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Black Hole–D-Brane Correspondence: An Example

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We explore the connection between D-branes and black holes in one particular case: a D3-brane compactified to four dimensions on T^6/Z_3. Using the D-brane boundary state description, we show the equivalence with a double extremal $N = 2$ black hole solution of four-dimensional supergravity.

1. INTRODUCTION

The lack of a statistical mechanical theory of black hole thermodynamics and the closely related problem of the black hole information paradox are longstanding fundamental questions which can now be precisely addressed. Explicit calculations are presently available due to the recent progress in nonperturbative aspects of string theory (see ref. 1 for a summary and references).

The idea of relating black holes to elementary string states is based on their common property of having a large degeneracy of states. However, while the entropy of a Schwarszchild black hole is proportional to the square of its mass, the logarithm of the degeneracy of elementary string states depends linearly on the mass of the states. It was suggested that this discrepancy is due to the large mass renormalization suffered by the string states due to quantum corrections, and thus could be avoided by BPS states in superstring theories. Following the analogy, the BPS condition on the states should correspond to the extremal condition on Reissner–Nordström black holes.

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A key step in the recent developments was the realization that in addition to the states described by string fluctuations, there are also soliton states in string theory, D-branes. The main advantage of using D-branes instead of perturbative string states is that the event horizon of the corresponding black hole is nonsingular and has finite area. Thus the entropy for these black holes can be computed unambiguously, and can be compared with the corresponding microscopic answer obtained from the counting of states of the D-brane. The two calculations turn out to be in exact agreement, including the overall numerical factor. Explicit calculations have been performed in many classes of black holes which can be compared to different configurations of D-branes. This result was obtained initially for a five-dimensional extremal black hole, and was later extended to five-dimensional rotating black holes, slightly nonextremal five-dimensional black holes, four-dimensional extremal, and slightly nonextremal black holes. The five-dimensional case was considered first since one only needs three nonzero charges to obtain an extremal black hole with regular horizon in toroidal compactifications. In four dimensions one needs four nonzero charges. For Calabi–Yau compactifications not all the results of the toroidal case hold. In particular, four different charges are no longer needed in four dimensions. Another characteristic of Calabi–Yau compactifications is that single D-brane black holes are nonsingular. This is because the brane is wrapped on a topologically nontrivial manifold and therefore can intersect itself, thus avoiding the necessity of having different branes in toroidal compactifications.

In this contribution we will explore the connection between D-branes and black holes in one particular case. We will explicitly show how the analogy can be carried through for a D3-brane compactified to four dimensions on T^6/Z_3 by providing the evidence that supports its identification with a double extremal $N = 2$ black hole in four dimensions. In Section 2 we summarize the boundary state description of a D3-brane wrapped on a 3-cycle of the T^6/Z_3 orbifold, which was originally introduced in ref. 2. We also recall the requirement imposed by the BPS condition, namely that the cancellation of the force between two identical D-branes in relative motion is due to the exchange of the $N = 2$ graviton multiplet containing the graviton and the graviphoton. This suggests that the classical solution corresponding to this configuration is a Reissner–Nordström black hole. In Section 3 we introduce the four-dimensional double extremal black hole solution of $N = 2$ supergravity obtained by compactifying ten-dimensional type IIB supergravity on a Calabi–Yau threefold. We also show in this section how the correspondence between this solution and the D3-brane boundary state description can be established [3].

2. D3-BRANES ON ORBIFOLDS

Let us consider a system of two D-branes in a type II superstring theory compactified down to four dimensions in the interesting case of the Z_3 orbifold, which breaks the supersymmetry down to $N = 2$ (the branes further break it to $N = 1$) [4, 5]. This section is based on refs. 2 and 5, where detailed calculations can be found.

The dynamics of these D-branes is determined by a one-loop amplitude which can be conveniently evaluated in the boundary state formalism [6, 7]. In particular, one can compute the force between two D-branes moving with constant velocity, extending Bachas' result [8] to compactifications breaking some supersymmetry [2]. This will be the key object to establish the D-brane–black hole correspondence. Analyzing the large-distance behavior of the interaction and its velocity dependence, it is possible to read the charges with respect to the massless fields and relate the various D-brane configurations to known solutions of the 4-dimensional low-energy effective supergravity.

The amplitude for two D-branes moving with velocities $V_1 = \tanh v_1$, $V_2 = \tanh v_2$ (say along 1) and transverse positions $\overline{\bar{Y}}_1$, $\overline{\bar{Y}}_2$ (along 2, 3), namely

$$\mathcal{A} = \int_0^\infty dl \sum_s \langle B, V_1, \overline{\bar{Y}}_1 | e^{-lH} | B, V_2, \overline{\bar{Y}}_2 \rangle_s \tag{1}$$

is just a tree-level propagation between the two boundary states which are defined to implement the boundary conditions specifying the branes. The time is measured along the length of the cylinder l. There are two sectors, RR and NSNS, corresponding to periodicity and antiperiodicity of the fermionic fields around the cylinder, and after the GSO projection there are four spin structures, R$\pm$ and NS$\pm$, corresponding to all the possible periodicities of the fermions on the covering torus.

Let us consider a D-particle in four-dimensional spacetime. In the static case, the 0-brane has Neumann boundary conditions in time and Dirichlet in space. The velocity twists the 0–1 directions and gives them rotated boundary conditions. The moving boundary state is most simply obtained by boosting the static one with a negative rapidity $v = v_1 - v_2$ [9],

$$|B, V, \overline{\bar{Y}}\rangle = e^{-ivJ^{01}} |B, \overline{\bar{Y}}\rangle$$

In the large-distance limit $b \to \infty$ only worldsheets with $l \to \infty$ will contribute, and momentum or winding in the compact directions can be safely neglected since they correspond to massive subleading components.

The moving-boundary states

$$|B, V_1, \bar{\vec{Y}}_1\rangle = \int \frac{d^3\bar{\vec{k}}}{(2\pi)^3}\, e^{i\bar{\vec{k}}\cdot\bar{\vec{Y}}_1}|B, V_1\rangle \otimes |k_B\rangle$$

$$|B, V_2, \bar{\vec{Y}}_2\rangle = \int \frac{d^3\bar{\vec{q}}}{(2\pi)^3}\, e^{i\bar{\vec{q}}\cdot\bar{\vec{Y}}_2}|B, V_2\rangle \otimes |q_B\rangle$$

can carry only space-time momentum in the boosted combinations to $k_B^\mu = (\sinh v_1 k^1, \cosh v_1 k^1, \vec{k}_T)$ and $q_B^\mu = (\sinh v_2 q^1, \cosh v_2 q^1, \vec{q}_T)$. Notice that because of their nonzero velocity, the branes can also transfer energy, and not only momentum as in the static case.

Integrating over the bosonic zero modes and taking into account momentum conservation ($k_B^\mu = q_B^\mu$), we find that the amplitude factorizes into a bosonic and a fermionic piece:

$$\begin{aligned}\mathcal{A} &= \frac{1}{\sinh v}\int_0^\infty dl \int \frac{d^2\vec{k}_T}{(2\pi)^2}\, e^{i\vec{k}\cdot\vec{b}} e^{-q_B^2 l/2} \sum_s Z_B Z_F^s \\ &= \frac{1}{\sinh v}\int_0^\infty \frac{dl}{2\pi l}\, e^{-b^2/2l} \sum_s Z_B Z_F^s \end{aligned} \tag{2}$$

with $Z_{B,F} = \langle B, V_1|e^{-lH}|B, V_2\rangle^s_{B,F}$. From now on, $X_\mu \equiv X^\mu_{osc}$.

It is very convenient to group the fields into pairs,

$$X^\pm = X^0 \pm X^1 \to \alpha_n, \qquad \beta_n = a_n^0 \pm a_n^1$$

$$X^i, X^{i*} = X^i \pm iX^{i+1} \to \beta_n^i, \qquad \beta_n^{i*} = a_n^i \pm i a_n^{i+1}, \qquad i = 2, 4, 6, 8$$

$$\chi^{A,B} = \psi^0 \pm \psi^1 \to \chi_n^{A,B} = \psi_n^0 \pm \psi_n^1$$

$$\chi^i, \chi^{i*} = \psi^i \pm i\psi^{i+1} \to \chi_n^i, \qquad \chi_n^{i*} = \psi_n^i \pm i\psi_n^{i+1}, \qquad i = 2, 4, 6, 8$$

with the commutation relations $[\alpha_m, \beta_{-n}] = -2\delta_{mn}$, $[\beta_m^i, \beta_{-n}^{i*}] = 2\delta_{mn}$, to $\{\chi_m^A, \chi_{-n}^B\} = -2\delta_{mn}$, $\{\chi_m^i, \chi_n^{i*}\} = 2\delta_{mn}$. For the RR zero modes, which satisfy a Clifford algebra and are thus proportional to Γ-matrices, $\psi_o^\mu = i\Gamma_\mu/\sqrt{2}$, $\tilde{\psi}_o^\mu = i\tilde{\Gamma}^\mu/\sqrt{2}$, one can construct similarly the creation–annihilation operators

$$a, a^* = \frac{1}{2}(\Gamma^0 \pm \Gamma^1), \qquad b^i, b^{i*} = \frac{1}{2}(-i\Gamma^i \pm \Gamma^{i+1})$$

satisfying the usual algebra $\{a, a^*\} = \{b^i, b^{i*}\} = 1$ (and similarly for tilded operators). In this way, any rotation or boost will reduce to a simple phase transformation on the modes. In fact, for an orbifold rotation ($g_a = e^{2\pi i z_a}$) one has

$$\beta_n^a \to g_a \beta_n^a, \qquad \chi_n^a \to g_a \chi_n^a, \qquad b^a \to g_a b^a$$

$$\beta_n^{a*} \to g_a^* \beta_n^{a*}, \qquad \chi_n^{a*} \to g_a^* \chi_n^{a*}, \qquad b^{a*} \to g_a^* b^{a*} \tag{3}$$

whereas for a boost of rapidity v,

$$\alpha_n \to e^{-v} \alpha_n, \qquad \chi_n^A \to e^{-v} \chi_n^A, \qquad a \to e^{-v} a$$

$$\beta_n \to e^{v} \beta_n, \qquad \chi_n^B \to e^{v} \chi_n^B, \qquad a^* \to e^{v} a^* \tag{4}$$

The boundary state which solves the boundary conditions can be factorized into a bosonic and a fermionic part; the latter can be further split into zero-mode and oscillator parts, and finally

$$|B\rangle = |B\rangle_b \otimes |B_0\rangle_F \otimes |B_{\rm osc}\rangle_F$$

Let us now look at the internal coordinates. An orbifold compactification can be obtained by identifying points in the compact part of spacetime which are connected by discrete rotations $g = \exp(2\pi i \Sigma_a z_a J_{aa+1})$ on some of the compact pairs X^a, χ^a, $a = 4, 6, 8$. In order to preserve at least one supersymmetry, one has to impose $\Sigma_a z_a = 0$.

Three cases can be considered: toroidal compactification on T_6 ($N = 8$ SUSY, $z_4 = z_6 = z_8 = 0$) and orbifold compactification on $T_2 \otimes T_4/Z_2$ ($N = 4$ SUSY, $z_4 = -z_6 = \frac{1}{2}$, $z_8 = 0$) and T_6/Z_3($N = 2$ SUSY, z_4, $z_6 = \frac{1}{3}, \frac{2}{3}$, $z_8 = -z_4 - z_6$).

The spectrum of the theory now contains additional twisted sectors, in which periodicity is achieved only up to an element of the quotient group Z_N. These twisted states exist at fixed points of the orbifold, and can thus occur only for 0-branes localized at one of the fixed points. We will not discuss this case here (see ref. 2).

Finally, in all sectors, one has to project onto invariant states to get the physical spectrum of the theory which is invariant under orbifold rotations. In particular, the physical boundary state is given by the projection $|B_{\rm phys}\rangle = \Sigma_k |B, g^k\rangle / N$, in terms of the twisted boundary states $|B, g^k\rangle = g^k |B\rangle$

Let us now concentrate in a particular 3-brane configuration. In the static case, we take Neumann boundary conditions for time, Dirichlet for space, and mixed for each pair of compact directions, say Neumann for the a directions and Dirichlet for the $a + 1$ directions.

The boundary state has to satisfy in the compact directions the following conditions:

$$(\beta_n^a + \beta_{-n}^{a*})|B\rangle_B = 0, \qquad (\beta_n^{a*} + \beta_{-n}^a)|B\rangle_B = 0$$

$$(\chi_n^a + i\eta\tilde{\chi}_{-n}^{a*})|B_{\rm osc}, \eta\rangle_F = 0, \qquad (\chi_n^{a*} + i\eta\tilde{\chi}_{-n}^a)|B_{\rm osc}, \eta\rangle_F = 0$$

$$(b^a + i\eta\tilde{b}^{a*})|B_0, \eta\rangle_F = 0, \qquad (b^{a*} + i\eta\tilde{b}^a)|B_0, \eta\rangle_F = 0$$

We define the spinor vacuum $|0\rangle \otimes |\tilde{0}\rangle$ such that $b^a|0\rangle = \tilde{b}^a|\tilde{0}\rangle = 0$. However, the boundary state is not invariant under orbifold rotations, under which the modes of the fields transform as in Eq. (3) and the spinor vacuum as $|0\rangle \otimes |\tilde{0}\rangle \to g_a|0\rangle \otimes |\tilde{0}\rangle$. This was expected since a Z_N rotation mixes two directions with different boundary conditions, and thus the corresponding closed string state does not need to be invariant under Z_N rotations. One finds for the compact part of the twisted boundary state

$$
\begin{aligned}
|B, V, g_a\rangle_B &= \exp\left\{ -\frac{1}{2} \sum_{n>0} (g_a^2 \beta_{-n}^a \beta_{-n}^a + g_a^{*2} \beta_{-n}^{a*} \beta_{-n}^{a*}) \right\} |0\rangle \\
|B_{osc}, V, g_a, \eta\rangle_F &= \exp\left\{ \frac{i\eta}{2} \sum_{n>0} (g_a^2 \chi_{-n}^a \tilde{\chi}_{-n}^a + g_a^{*2} \chi_{-n}^{a*} \tilde{\chi}_{-n}^{a*}) \right\} |0\rangle \qquad (5) \\
|B_0, V, g_a, \eta\rangle_{RR} &= g_a \exp\left\{ -i\eta g_a^{*2} b^{a*} \tilde{b}^{a*} \right\} |0\rangle \otimes |\tilde{0}\rangle
\end{aligned}
$$

After the GSO projection, the total partition functions for a given relative angle w_a turn out to be

$$
Z_B = 16i \sinh v q^{1/3} f(q^2)^4 \frac{1}{\vartheta_1(iv/\pi|2il)} \prod_a \frac{\sin \pi w_a}{\vartheta_1(w_a|2il)} \qquad (6)
$$

$$
\begin{aligned}
Z_F &= q^{-1/3} f(q^2)^{-4} \left\{ \vartheta_2\left(i\frac{v}{\pi} |2il \right) \prod_a \vartheta_2(w_a|2il) \right. \\
&\quad \left. - \vartheta_3\left(i\frac{v}{\pi} |2il \right) \prod_a \vartheta_3(w_a|2il) + \vartheta_4\left(i\frac{v}{\pi} |2il \right) \prod_a \vartheta_4(w_a|2il) \right\} \qquad (7) \\
&\sim \begin{cases} V^4, & w_a = 0 \\ V^2, & w_a \neq 0 \end{cases} \qquad (8)
\end{aligned}
$$

Recall that to obtain the invariant amplitude, one has to average over all possible angles w_a.

In the large-distance limit $l \to \infty$, explicit results with exact dependence on the rapidity can be obtained from the above expression and compared to a field theory computation. One finds the following behaviors, according to the compactification scheme:

$$
\mathcal{A}(w_a) \sim 4 \prod_a \cos \pi w_a \cosh v - \cosh 2v - \sum_a \cos 2\pi w_a
$$

$$
\mathcal{A} \sim \begin{cases} 4 \cosh v - \cosh 2v - 3 \sim V^4, & T_2 \otimes T_4/Z_2, \quad T_6 \\ \cosh v - \cosh 2v \sim V^2, & T_6/Z_3 \end{cases} \qquad (9)
$$

Let us now compare the large-distance interactions of the two moving branes found from the string formalism with the field theory results. At large distances we look for the Feynman graphs representing the exchange of massless particles which can be either scalar, vector, or graviton. Since we consider two branes of the same nature, the scalar and the graviton give attraction, while the vector gives repulsion.

The net result in the static case is zero, since the branes are BPS states, and this is what is obtained from the Riemann identity in the string formalism [10]. But when the velocity is different from zero, the various contributions are unbalanced. By comparing the velocity dependence with what is obtained from Feynman graphs one can tell which kind of particles are actually coupled to the branes in the various compactifications.

We treat the branes as spinless particles of mass and charge equal to 1. The exchange of a scalar gives then

$$S = \frac{1}{k_{\perp}^2} \tag{10}$$

where k is the momentum transfer between the two branes. In the so-called eikonal approximation in which the branes go straight (which is the standard setting for describing the branes' interaction at nonsmall distances), k has only space components and is orthogonal to $\bar{V}$.

The vector is coupled to the current, which in the eikonal approximation is proportional to the momentum, $J^{\mu}(V) \equiv (\cosh(v), \sinh(v))$. Note that $J^{\mu}k_{\mu} = 0$. Taking one of the branes at rest, the vector exchange is

$$\mathcal{V} = J^{\mu}(V)J_{\mu}(0)\,\frac{1}{k_{\perp}^2} = -\frac{\cosh(v)}{k_{\perp}^2} \tag{11}$$

The graviton is coupled to the brane's energy-momentum tensor $T^{\mu\nu} = J^{\mu}J^{\nu}$. Therefore the graviton exchange in d dimensions is

$$\begin{aligned}\mathcal{G} &= 2(T^{\mu\nu}(V) - \frac{\eta^{\mu\nu}}{d-2}\,T^{\rho\sigma}(V)\eta_{\rho\sigma})T_{\mu\nu}(0)\,\frac{1}{k_{\perp}^2}\\ &= \frac{\cosh(2v) + (d-4)/(d-2)}{k_{\perp}^2}\end{aligned} \tag{12}$$

Thus the nature of the various contributions to the branes' interaction can be read from the rapidity dependence of the $l \to \infty$ limit of the amplitude (7), and is the following for $d = 4$:

$$\begin{aligned}4\cosh v - \cosh 2v - 3 &\Leftrightarrow N = 8 \text{ grav. multiplet}\\ \cosh v - \cosh 2v &\Leftrightarrow N = 2 \text{ grav. multiplet}\end{aligned} \tag{13}$$

In the second case, the two branes interact through the exchange of the RR vector and the universal graviton with no scalar exchange. In terms of the $N = 2$ SUSY theory these systems couple only to the graviton and its $N = 2$ partner, the graviphoton. From the pattern of cancellation [11] these branes seem to correspond to classical extremal Reissner–Nordström black holes. We present the evidence to support this conjecture in the next section.

3. $N = 2$ BLACK HOLE SUPERGRAVITY SOLUTIONS

BPS-saturated solutions of four-dimensional $N = 2$ supergravity coupled to $N = 2$ vector multiplets have been discussed in many recent papers. The simplest class of solutions is given by the double extremal $N = 2$ black holes with nonvanishing electric and magnetic charges. For this type of solution the values of the scalar moduli fields which follow from a minimization of the $N = 2$ central charge take constant values over the entire spacetime. In more general cases of nonconstant moduli, the internal space does not decouple from the four-dimensional spacetime. In particular in static extremal $N = 2$ black hole solutions the vector multiplet moduli vary over the uncompactified space and one can argue that special or singular points in the internal space are related to special or singular points in spacetime (like horizons or curvature singularities).

The concept of a *double-extremal* black hole was introduced in ref. 12. Nonextremal black holes have two horizons. When they coincide the black hole is called extremal. As solutions of supergravities, the mass of the extremal black hole depends on moduli as well as on quantized charges. Double-extremal black holes are extremal, supersymmetric black holes with the extremal value of the ADM mass equal to the Bertotti–Robinson mass. They have constant moduli both for vector multiplets as well as for hypermultiplets, but the electric and magnetic charges in each gauge group are unconstrained. We will obtain a four-dimensional double-extremal black hole by compactifying an exact solution of type IIB supergravity in 10 dimensions on a 3-cycle of the generic threefold $\mathcal{M}_3^{CY}$.

Let us start by considering the field equations of type IIB supergravity in 10 dimensions, namely

$$R_{MN} = T_{MN} \tag{14}$$

$$\nabla_M F_{(5)}^{MABCD} = 0 \leftarrow F^{(5)}_{G_1 \ldots G_5} = \frac{1}{5!} \epsilon_{G_1 \ldots G_5 H_1 \ldots H_5} F^{H_1 \ldots H_5} \tag{15}$$

where $T_{MN} = 1/(2 \cdot 4!) F^{(5)}_{M\ldots} F^{(5)}_{N\ldots}$ is the traceless energy-momentum tensor of the RR 4-form $A_{(4)}$ to which the 3-brane couples and $F_{(5)}$ is the corresponding self-dual field strength. The tracelessness of T_{MN} and the absence of couplings to the dilaton (see, for instance, ref. 13) allow for zero-curvature solutions in 10 dimensions.

For the metric we make a block-diagonal ansatz. We take for the four-dimensional part $g^{(4)}_{\mu\nu}$ the extremal RN black hole solution, which depends only on the corresponding noncompact coordinates x^μ. The Ricci-flat compact part depends only on the internal coordinates y^a (this corresponds to choosing the unique Ricci-flat Kähler metric on $\mathcal{M}_3^{CY}$),

$$ds^2 = g^{(4)}_{\mu\nu}(x)\, dx^\mu\, dx^\nu + g^{(6)}_{ab}(y)\, dy^a\, dy^b \tag{16}$$

In general, the compact components of the metric depend on the noncompact coordinates x^μ through the moduli which parametrize the deformations of the Kahler class or the complex structure. In type IIB compactifications such moduli belong to hypermultiplets and vector multiplets. In our case, however, where the Hodge number $h^{(2,1)} = 0$, there are no vector multiplet scalars that would couple nonminimally to the gauge fields, and the hypermultiplet scalars can be set to zero since they do not couple to the unique gauge field, namely the graviphoton [therefore $g_{ab}(x, y) = g_{ab}(y)$].

The 5-form field strength can be generically decomposed in the basis of all the harmonic 3-forms of the CY manifold $\Omega^{(i,j)}$

$$F_{(5)}(x, y) = F^0_{(2)}(x) \wedge \Omega^{(3,0)}(y) + \sum_{k=1}^{h^{(2,1)}} F^k_{(2)}(x) \wedge \Omega^{(2,1)}_k(y) + \text{c.c.} \tag{17}$$

In the case at hand, however, only the graviphoton $F^0_{(2)}$ appears in the general ansatz (17), without any additional vector multiplet field strength $F^k_{(2)}$. We conveniently normalize

$$F_{(5)}(x, y) = \frac{1}{\sqrt{2}} F^0_{(2)}(x) \wedge (\Omega^{(3,0)} + \Omega^{(0,3)}) \tag{18}$$

Notice that this same ansatz is consistent for any double-extremal solution even for a more generic CY.

With these ansätze, Eq. (14) reduces to the usual four-dimensional Einstein equation with a graviphoton source. The compact part is identically satisfied. The four-dimensional Lagrangian is obtained by carrying out the explicit integration over the CY. Choosing an appropriate normalization for $\Omega^{(3,0)}$ and $\Omega^{(0,3)}$ such that $\|\Omega^{(3,0)}\|^2 = V^2_{D3}/V_{CY}$ (since the volume of the corresponding 3-cycle is precisely the volume V_{D3} of the wrapped 3-brane) one has [($z^a = 1/\sqrt{2}(y^a + iy^{a+1})$ and $d^6y = id^3z d^3\bar{z}$]

$$\int_{\mathrm{CY}} d^6y \sqrt{g_{(6)}} = V_{\mathrm{CY}}, \qquad i\int_{\mathrm{CY}} \Omega^{(3,0)} \wedge \Omega^{(0,3)} = V_{\mathrm{D3}}^2 = \int_{\mathrm{CY}} d^6y \sqrt{g_{(6)}} \|\Omega^{(3,0)}\|^2 \tag{19}$$

and then

$$\mathcal{S} = \frac{1}{2k_{(4)}^2} \int d^4x \sqrt{g_{(4)}} \left(R_{(4)} - \frac{1}{2 \cdot 2!} \operatorname{Im} \mathcal{N}_{00} F^0_{\mu\nu} F^{0\mu\nu} \right) \tag{20}$$

where $k_{(4)}^2 = k_{(10)}^2/\mathrm{V}_{\mathrm{CY}}$ and $\operatorname{Im} \mathcal{N}_{00} = V_{\mathrm{D3}}^2/V_{\mathrm{CY}}$. In the general case, Eq. (17) integration over the CY gives rise to a gauge field kinetic term of the standard form: $\operatorname{Im} \mathcal{N}_{\Lambda\Sigma} F^\Lambda F^\Sigma + \operatorname{Re} \mathcal{N}_{\Lambda\Sigma} F^\Lambda {}^*F^\Sigma$, where $\Lambda, \Sigma = 0, 1, \ldots, h^{(1,2)}$. As well known (from now on $F^0_{(2)} \equiv F$), the four-dimensional Maxwell–Einstein equations of motion following from this Lagrangian admit the extremal RN black hole solution (in coordinates in which the horizon is located at $r = 0$)

$$g_{00} = -\left(1 + \frac{\kappa_{(4)} M}{r}\right)^{-2}, \qquad g_{mn} = \left(1 + \frac{\kappa_{(4)} M}{r}\right)^{2} \tag{21}$$

$$F_{m0} = \kappa_{(4)} e_0 \frac{x^m}{r^3} \left(1 + \frac{\kappa_{(4)} M}{r}\right)^{-2}, \qquad F_{mn} = \kappa_{(4)} g_0 \epsilon_{mnp} \frac{x^p}{r^3}$$

where $m, n, p = 1, 2, 3$. The extremality condition is $M^2 = (e^2 + g^2)/4$, where for later convenience we parametrize the solution with

$$M = \frac{\hat{\mu}}{4}, \qquad e = e_0 \sqrt{\frac{V_{\mathrm{D3}}^2}{V_{\mathrm{CY}}}} = \frac{\hat{\mu}}{2} \cos\alpha, \qquad g = g_0 \sqrt{\frac{V_{\mathrm{D3}}^2}{V_{\mathrm{CY}}}} = \frac{\hat{\mu}}{2} \sin\alpha \tag{22}$$

The parameter $\hat{\mu}$ is related to the 3-brane tension μ through $\hat{\mu} = \sqrt{V_{\mathrm{D3}}^2/V_{\mathrm{CY}}}\,\mu$, and the angle α depends on the way the 3-brane is wrapped on the CY. Notice that the charges with respect to the gauge field A^μ are e_0 and g_0, but since the kinetic term and, correspondingly, the propagator of A^μ are not canonically normalized, the effective couplings appearing in a scattering amplitude are rather e and g, which indeed satisfy the usual BPS condition. Further, at the quantum level, e and g are quantized as a consequence of Dirac's condition $eg = 2\pi n$; correspondingly, the angle α can take only discrete values and this turns out to be automatically implemented in the compactification [14].

This ends the field theory side of the computation. Let us now compare with the microscopic string theory description of the same black hole introduced in the previous section.

The interaction between two D3-branes compactified on T^6/Z_3 in relative motion, Eqs. (7) and (9) for large impact parameters, can be rewritten as

$$\mathcal{A} = \frac{\hat{\mu}^2}{4} (\cosh v - \cosh 2v) \int dt \, \Delta_3(r) \tag{23}$$

where $\Delta_3(r)$ is the three-dimensional Green function, $r = \sqrt{b^2 + \sinh^2 vt^2}$, and b is the impact parameter. This four-dimensional configuration comes from the effective action

$$\mathcal{S} = \int d^4x \sqrt{g} \left(R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2 \cdot 2!} e^{-a\phi} F_{(2)}^2 \right) \tag{24}$$

where $a = 0$ for the RN black hole and $a \neq 0$ for the 0-brane. We concentrate on the first case, for which the general electric extremal solution of this Lagrangian is [16]

$$ds^2 = -H(r)^{-2} dt^2 + H(r)^2 d\vec{x} \cdot d\vec{x}, \qquad \phi = 0, \qquad A_0 = 2H(r)^{-1} \tag{25}$$

where $H(r)$ satisfies the three-dimensional Laplace equation and can be taken to be of the form $H(r) = 1 + k\Delta_3(r)$. The relevant asymptotic long-range fields are thus

$$h_{00} = 2k \, \Delta_3(r), \qquad A_0 = 2k \, \Delta_3(r)$$

Comparing with Eq. (23), we find that the RN solution corresponds to $k = \hat{\mu}/4$.

An equivalent way of analyzing this configuration and providing more elements to identify the D3-brane with the general RN $\times$ CY solution discussed before is to compute one-point functions $\langle \Psi \rangle = \langle \Psi | B \rangle$ of the massless fields of supergravity and compare them with the linearized long-range fields of the supergravity RN black hole solution (21). This second method presents the advantage of yielding direct information on the coulpings with the massless fields of the low-energy theory.

Let us consider the case in which the internal directions of the D3-brane form an arbitrary common angle θ_0 with the X^a directions in each of the 3-planes X^a, X^{a+1} (actually, we could have chosen three different angles in the 3-planes, but only their sum will be relevant). The Z_3 projection is implemented by $|B\rangle = \frac{1}{3}\Sigma_{\{\Delta\theta\}}|B_3(\theta = \Delta\theta + \theta_0)\rangle$, where the sum is over $\Delta\theta = 0$, $2\pi/3$, $4\pi/3$. It is obvious from this formula that $|B\rangle$ is a periodic function of the parameter θ_0 with period $2\pi/3$. Therefore, the physically distinct values of θ_0 are in $[0, 2\pi/3]$ and define a one-parameter family of Z_3-invariant boundary states, corresponding to all the possible harmonic 3-forms on

T^6/Z_3, as we will see. Notice that requiring a fixed finite volume V_{D3} or the 3-cycle on which the D3-brane is wrapped implies discrete values for θ_0 [14]. The compactification process restricts the momenta entering the Fourier decomposition of $|B\rangle$ to belong to the momentum lattice of T^6/Z_3. Since the massless supergraviton states $|\Psi\rangle$ carry only spacetime momentum, the compact part of the boundary state will contribute a volume factor which turns the 10-dimensional D3-brane tension $\mu = \sqrt{2\pi}$ into the four-dimensional black hole charge $\hat{\mu} = \sqrt{V_{D3}^2/V_{CY}}\mu$ [14], and some trigonometric functions of θ_0 to be discussed below.

Using the technique of ref. 15, the relevant one-point functions on $|B_3(\theta)\rangle$ for the graviton and 4-form states $|h\rangle$ and $|A\rangle$ can be computed and one finds, by comparing with the boundary state result, that the electric and magnetic charges are

$$e = \frac{\hat{\mu}}{2}\cos 3\theta_0, \qquad g = \frac{\hat{\mu}}{2}\sin 3\theta_0 \tag{26}$$

Comparing with Eq. (22), one obtains $\alpha = 3\theta_0$ and therefore the ratio between e and g depends on the choice of the 3-cycle, as anticipated. Also, as explained, only discrete values of θ_0 naturally emerge requiring a finite volume.

Further evidence for the identifications (26) comes from the computation of the electromagnetic phase shift between two of these configurations with different θ_0's, call them $\theta_{1,2}$. Since the four-dimensional electric and magnetic charges of the two black holes are then different, there should be both an even and an odd contribution to the phase shift coming from the corresponding RR spin structures. Indeed, one correctly finds [14]

$$\begin{aligned} \mathcal{A}_{\text{even}} &\sim \frac{\hat{\mu}^2}{4}\cos 3(\theta_1 - \theta_2) = e_1 e_2 + g_1 g_2, \\ \mathcal{A}_{\text{odd}} &\sim \frac{\hat{\mu}^2}{4}\sin 3(\theta_1 - \theta_2) = e_1 g_2 - g_1 e_2 \end{aligned} \tag{27}$$

Therefore the asymptotic gravitational and electromagnetic fields of the RN black hole, Eqs. (21), are correctly reproduced. This confirms that our boundary state describes a D3-brane wrapped on T^6/Z_3, falling in the class of regular four-dimensional RN double-extremal black holes obtained by wrapping the self-dual D3-brane on a generic CY threefold. This boundary state encodes the leading-order couplings to the massless fields of the theory, and allows the direct determination of their long-range components, falling off like $1/r$ in four dimensions. The subleading post-Newtonian corrections to these fields arise instead as open string higher loop corrections, corresponding to string worldsheets with more boundaries; from a classical field theory point of view, this is the standard replica of the source in the tree-level perturbative evaluation of a nonlinear classical theory.

To conclude, let us comment that one could interpret the Z_3-invariant boundary state as describing the three-D3-brane superposition at angles $(2\pi/3)$ in a T^6 compactification. As illustrated in ref. 17, such an intersection preserves precisely 1/8 supersymmetry, as a single D3-brane does on T^6/Z_3. For toroidal compactification this is not enough, of course, because at least four intersecting D3-branes are needed in order to get a regular solution [18].

Finally, since this extremal RN configuration is constructed by a single D3-brane, the question naturally arises of understanding the microscopic origin of its entropy.

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